Math 105, Assignment 1 **Due 2015-01-16, 4:00 pm**

In the following problems you are expected to justify your answers unless stated otherwise. Answers without any explanation will be given a mark of zero. The assignment needs to be in my hand before I leave the lecture room or you will be given a zero on the assignment!

1. (a) What is the equation of the plane Q, that passes through the point $(1, 2, -4)$ and parallel to the plane $P : 2x - y + 6z = 4$?

   (b) Determine all the vectors of length 5 perpendicular to P.

2. Determine the domain of

$$f(x, y) = \frac{\log(y + 1)}{(x^2 - y^2)(1 - \sin x)}.$$

   Sketch it on the xy-plane.

3. Sketch the level curve at $z = 0$ of

$$z = f(x, y) = (x^2 - 2x + 4y^2 - 3)\log(x^2 + y^2).$$

4. Sketch the $xy-, xz-, yz-$ traces of

$$4x^2 - 9(y - 1)^2 - z = 0.$$

   Give a very rough sketch of

$$z = f(x, y) = 4x^2 - 9(y - 1)^2.$$

5. Let $0 \neq \vec{x}, \vec{y}$ be vectors in $\mathbb{R}^n$, the goal of this question is to prove the **triangle inequality** for vectors:

$$\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$$

   (a) Define a function

$$P(t) = \|t\vec{x} + \vec{y}\|^2.$$

   Using the fact that $\|\vec{x}\|^2 = \vec{x} \bullet \vec{x}$, show:

$$P(t) = \|\vec{x}\|^2 t^2 + 2(\vec{x} \bullet \vec{y})t + \|\vec{y}\|^2$$

   (b) Show that the minimum of $P(t)$ is

$$\frac{\|\vec{x}\|^2\|\vec{y}\|^2 - |\vec{x} \bullet \vec{y}|^2}{\|\vec{x}\|^2}.$$

   Since $P(t) \geq 0$, we can deduce that

$$|\vec{x} \bullet \vec{y}| \leq \|\vec{x}\|\|\vec{y}\|.$$

   This is called the **Cauchy-Schwartz** inequality.

(c) Use (a) and (b) to deduce that

$$\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|.$$

You many assume triangle inequality for real numbers i.e. $|x + y| \leq |x| + |y|$ for all $x, y \in \mathbb{R}$. **Hint**: First show that

$$|P(t)| \leq (t\|\vec{x}\| + \|\vec{y}\|)^2,$$

then set $t = 1$